Set-1

Code No. 23J131 /NC/MAT

Nizam College (Autonomous)

Faculty of Science

B.SC. I- Semester Examinations, January - 2023

Mathematics : Paper-I

Time : 3 Hours

Max. Marks : 80

Section – A

Answer any EIGHT of the following questions. [8 x 4 = 32]

1. Solve $(2ax + x^2)\frac{dy}{dx} = a^2 + 2ax$
2. Solve $\frac{dy}{dx} + \frac{y}{\sqrt{1-x}\,\sqrt{x}} = 1 - \sqrt{x}$
3. Solve $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}$
4. Solve $yp^2 + (x - y)p - x = 0$
5. Solve $p = \tan\left(x - \frac{p}{1+p^2}\right)$
6. Solve $y = yp^2 + 2px$
7. Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$
8. Find the particular integral of the differential equation $y'' + y' + y = x^2$
9. Solve $y'' - y = 2e^x$
10. Solve $(x^2D^2 + 2xD - 12)y = 0$
11. Form the partial differential equation from :

    $z = xy + y\sqrt{x^2 - a^2} + b$
12. Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$

[P.T.O]

Section – B

II. Answer the following questions using internal choice.

13\. (a) (i) Solve $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$

(ii) Solve $\cos x dy = y(\sin x - y)\, dx$

[OR]

(b) (i) Solve $y\sin 2x dx - \ (1 + y^2 + cos^2 x)dy = 0$

(ii) Solve $\frac{dx}{y-z} = \frac{dy}{z-x} = \frac{dz}{x-y}$

14\. (a) (i) Solve $x^2p^2 - 2xyp + \ (2y^2 - x^2) = 0$

(ii) Solve $sinpxcosy = cos\ px\ siny + p$

[OR]

(b) Solve $y + px = p^2x^4$

15\. (a) (i) Solve $(D^2 + 1)y = xe^{2x}$

(ii) Solve $(D^4 + D^2)y = 3x^2 + 4sinx - 2cosx$

[OR]

(b) Solve the differential equation by the method of undetermined coeffi
$(D^2 - 2D - 8)y = 9xe^x + 10e^{-x}$

16\. (a) Using the method of variation of parameter, solve the following differ equation $y'' + 2y' + y = e^{-x} \log x$

[OR]

(b) (i) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

(ii) Form the partial differential ciruation by eliminating the arbitrary function 'f' form $z = xy + f(x^2 + y^2)$

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Code No. 23M131 /NC/MAT

Nizam College (Autonomous)
Faculty of Science
B.SC. I- Semester Examinations, May - 2023
Mathematics : Paper-I

Time : 3 Hours

Max. Marks : 80

Section – A

I. *Answer any EIGHT of the following questions.* 8 x 4 = 32]

1. Solve $(x+y)^2 \frac{dy}{dx} = a^2$
2. Solve $x\frac{dy}{dx} = y(\log y - \log x + 1)$
3. Solve $(x^3e^x - my^2)dx + mxydy = 0$
4. Solve $x^2p^2 + xyp - 6y^2 = 0$
5. Solve $(y - px)(p - 1) = p$
6. Solve $y = 2px + y^{n-1}p^n$
7. Solve $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$
8. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 5$
9. Solve $(D^2 - 2D + 5)y = e^{-x}$
10. Solve $(x^2D^2 + 2xD - 12)y = 0$
11. Form the partial differential Equation by eliminating the arbitrary function $z = xy + f(x^2 + y^2)$
12. Form the partial differential Equation by eliminating the arbitrary Constants from $z = ax^2 + bxy + cy^2$

Section – B

II. *Answer the following questions using internal choice.* [4 x 12 = 48]

13. (a) (i) Solve $(x - y)^2dx + 2xydy = 0$

(ii) Solve $(2x + 4y + 3)\frac{dy}{dx} = x + 2y + 1$

[OR]

(b) (i) Solve $x^2ydx - (x^3 + y^3)dy = 0$

(ii) Solve $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy = 0$

14. (a) Solve $y^2\log y = xpy + p^2$

[OR]

(b) (i) Solve $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$

(ii) Solve $p = \log(px - y)$

15. (a) (i) Solve $(D^3 + 1)y = \cos 2x$

(ii) Solve $(D^2 + 4D + 4)y = e^{2x} - e^{-2x}$

[OR]

(b) Solve the Differential Equation by the method of undetermined Coefficients : $(D^2 - 2D)y = e^x \sin x$.

16. (a) Using the method of variation of parameters, solve $y'' - 2y' + y = e^x \log x$

[OR]

(b) (i) Solve $px + qy = z$

(ii) Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$

Code No. 23D/131 /NC/MAT

Nizam College (Autonomous)

Faculty of Science

B.SC. I- Semester Examinations, December - 2023

Mathematics : Paper-I

(Differential Equations)

Time : 3 Hours

Max. Marks : 80

Section – A

I. *Answer any EIGHT of the following questions.* [8 x 4 = 32]

1. *Solve* $xdy - ydx = a(x^2 + y^2)dy$.
2. *Define linear equation.*
3. *Solve* $p^2 - 5p + 6 = 0$.
4. *Define an orthogonal trajectory.*
5. *Solve* $p = \log(px - y)$.
6. *Define Exact Differential Equation.*
7. *Solve* $(D^3 - 7D + 6)y = 0$.
8. *Find the value of* $\frac{1}{D^2+4}\cos 2x$.
9. *Find the Particular Integral* of $(D^4 - 1)y = \sin x$.
10. *Solve* $\frac{d^2y}{dx^2} = xe^x$.
11. *Define Legendere's equation.* ..
12. Define a Partial Differential equation with an example.

Section – B

II. *Answer the following questions.* [4 x 12 = 48]

13. (a) *Solve* $x^2ydx - (x^3 + y^3)dy = 0$.

[OR]

(b) *Solve* $x\frac{dy}{dx} - y = \log x$.

14. (a) *Solve* $y + px = p^2x^4$.

[OR]

(b) *Solve* $x = y + p^2$.

15. (a) *Solve* $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$.

[OR]

(b) *Solve* $(D^2 - 2D + 5)y = e^{2x}\sin x$.

16. (a) *Solve* $(D^2 + 2D + 1)y = e^{-x}\log x$ *by the method of variation of parameters.*

[OR]

(b) *Solve* $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$.

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Set-1

CODE NO. 23J/231/NC/MAT

NIZAM COLLEGE (AUTONOMOUS)
FACULTY OF SCIENCE
B.SC. II- SEMESTER EXAMINATIONS, JUNE – 2023
MATHEMATICS : PAPER - II
(DIFFERENTIAL AND INTEGRAL CALCULUS)

TIME: 3 HOURS

MAX. MARKS: 80

SECTION – A

Answer any EIGHT of the following questions. [8 x 4 = 32]

1. If $f(x,y) = y\cos(xy)$ then evaluate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$
2. If $u = \sin^{-1}\frac{x}{y}$ then find $\frac{\partial^2 u}{\partial x \partial y}$.
3. Define homogenous function and give two examples.
4. Find the total derivative of $f(x,y,z) = e^{xyz}$.
5. If $x^y = y^x$ then find $\frac{dy}{dx}$.
6. Expand $f(x,y) = e^y \log(1+x)$ in powers of x and y at $(0,0)$ by using Taylor's series.
7. Find the radius of curvature at origin of the curve $2x^4 + 2y^4 + 4x^2y + xy - y^2 + 2x = 0$
8. Find the center of curvature for the curve $y^2 = 4ax$ at the point $(a, 2a)$.
9. Find the envelope of the curve $y = mx + 2m^3$ where m is the parameter.
10. Find the length of the curve $y = x\sqrt{x}$ from $x = 0$ to $x = \frac{4}{3}$.
11. Find the surface area of a sphere of radius 'a'.
12. Find the volume of the solid of revolution generated by revolving the plane area bounded by curve $y = x^3$, $y = 0$, $x = 2$ about $X - axis$.

SECTION – B

Answer the following questions. [4 x 12 = 48]

13. (a) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $(x \neq y)$ then show that

   i. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

   ii. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$.

   [OR]

   (b) If $V = r^m$ where $r^2 = x^2 + y^2 + z^2$ then prove that $V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-2}$

14. (a) Find the maximum or minimum of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

   [OR]

   (b) Find $\frac{du}{dt}$ if $u = \tan^{-1}\frac{y}{x}$ given $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$.

15. (a) Find the equation of circle of curvature for the equation $x^3 + xy^2 - 6y^2 = 0$ at $(3,3)$.

   [OR]

   (b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a, b are parameters connected by the relation $a + b = c$, where c is constant.

16. (a) Find the whole length of the arc of the semi-cubical parabola $y^3 = ax^2$ from origin to the point $\left(\frac{a}{8}, \frac{a}{4}\right)$

   [OR]

   (b) Find the area of the surface of the solid generated by the revolution of an arc of curve $y = c\cosh\left(\frac{x}{c}\right)$ about X-axis.

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Set-1 Code No. 23J331 /NC/MAT

Nizam College (Autonomous)
Faculty of Science
B.SC. III- Semester Examinations, January - 2023
Mathematics : Paper-III
(Real Analysis)

Time : 3 Hours]

[Max. Marks : 80

Section – A

I. Answer any EIGHT of the following questions. [8 x 4 = 32]

1. Define convergent sequence.
2. Prove that the Sequence $S_n = \frac{3n-1}{n+2}$ is increasing and bounded above
3. State Geometric series. Test for convergence $\sum \frac{1}{2^n}$
4. Define continuity of a function at a point.
5. Find the values of $fog(0), gof(0), fog(2)$ and $gof(2)$ where $f(x)=\sqrt{4-x}$ for $x \le 4$ and $g(x)=x^2$ for all $x \in R$.
6. Find C of the function $f(x) = sinx$ on $[1,3]$ by using mean value theorem.
7. State the Cauchy's mean value theorem.
8. Write the n^{th} term of the sequence $1, -4, 9, -16, 25 \ldots\ldots\ldots\ldots$
9. Verify mean value theorem for $f(x) = x^2$ on $[-1,2]$.
10. If $f(x) = x$ on $[0,1]$ and $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ find Lower Riemann sum?
11. Define Upper and Lower Riemann sum.
12. Prove that $\sum_{n=1}^{\infty} \frac{1}{2^n} = 2$.

Section – B

II. Answer the following questions using internal choice. [4 x 12 = 48]

13. a) Every convergent sequence is bounded.

[OR]

b). Find $m \in z^+$ such that $\left|\frac{2n}{n+3} - 2\right| < \frac{1}{5}$ $\forall n \ge m$.

14. a) Let $f(x) = 2x^2 + 1$ for $x \in R$. Prove f is continuous on R by i) using the definition and ii) using the $\in -\delta$ property.

[OR]

b) If a function f is continuous on $[a, b]$ then it is uniformly continuous on $[a, b]$.

15. (a) If f is differentiable at a point a, then prove that f is continuous at a.

[OR]

(b) i) State and prove Rolles theorem.
ii) Verity the Rolles then $f(x) = x^2 - 6x + 8$ in $[2,4]$.

16 . (a) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that $U(f, P) - L(f, P) < \varepsilon$.

[OR]

(b) State and prove the fundamental theorem of Integral calculus.

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Code No. 23M331 /NC/MAT

Nizam College (Autonomous)
Faculty of Science
B.SC. III- Semester Examinations, May - 2023
Mathematics : Paper-III
(Real Analysis)

Time : 3 Hours

Max. Marks : 80

Section – A

I. Answer any EIGHT of the following questions.

[8 x 4 = 32]

1. Define sandwich theorem.
2. Prove that sequence $s_n = (-1)^n$ does not converge.
3. State Ratio test show that the sequence $s_n = \sin\frac{n\pi}{3}$ do not converges.
4. Prove every polynomial function $P(x) = a_0 + a_1 + a_2x^2 + \cdots\cdots + a_nx^n$ is continuous on R .
5. Let f and g be real-valued functions that are continuous at x_0 in R. Then prove that $f + g$ is continuous at x_0 .
6. Prove that the function $f(x) = 3x + 1$ is uniformly continuous on R bt directly verifying the $\in -\delta$ property.
7. State the Taylor's theorem.
8. State Languages mean value theorem.
9. Verify mean value theorem for the function $f(x) = \frac{1}{x}$ on [1.3] .
10. If $f(x) = x^2$ on [0,1] and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$, then find the Upper Riemann sum.
11. If f is integrable on $[a, b]$, then prove that $|f|$ is integrable on $[a, b]$.
12. Define Upper and Lower Riemann integral.

[P.T.O]

Section – B

II. Answer the following questions using internal choice. [4 x 12 = 48]

13. (a) i) Every converges sequence is bounded.

ii) Prove that the sequence $S_n = \frac{3n-1}{n-2}$ is increasing and bounded above.

[OR]

(b) State and prove $P - series\ test.$

14. (a) Show that $f(x) = \frac{1}{x^2}$ is not uniformly continuous on the set $(0, \infty)$.

[OR]

(b) If f is uniform continuous on s then f is continuous on s.

15. (a) If f is differentiable at a point a and g is differentiable at $f(a)$ then prove that the composite function gof is continuous at a and $(gof)^1(a) = g^1(f(a)).f^1(a)$.

[OR]

(b) i) State and prove Cauchy mean value theorem.

ii) Find 'c' by using Cauchy mean value theorem $f(x) = x^2, g(x) = x^3 in\ [1,2]$.

16. (a)) Let f be a bounded function on $[a, b]$. If P and Q are partitions of $[a, b]$ and $P \subseteq Q$, then prove that $L(f, P) \leq L(f, Q) \leq U(f, Q \leq U(f, P)$.

[OR]

(b) State and prove the fundamental theorem of Integral calculus.

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Code No. 23D/331 /NC/MAT

Nizam College (Autonomous)

Faculty of Science

B.SC. III- Semester Examinations, December - 2023

Mathematics : Paper-III

(Real Analysis)

Time : 3 Hours

Max. Marks : 80

Section – A

I. *Answer any EIGHT of the following questions.* [8 x 4 = 32]

1. Find the convergence of the sequence $< s_n >$ where $s_n = 73 + (-1)^n$.
2. Prove that every convergent sequence is bounded.
3. Prove that $\lim_{n\to\infty}\left[\sqrt{(n^2+n)} + n\right] = \frac{1}{2}$.
4. If f and g are real valued functions at x_0 then $f+g$ is continuous at x_0.
5. Prove that $x = \cos x$, for some x in $\left(0, \frac{\pi}{2}\right)$.
6. Find the limit of $f(x)$, where $f(x) = \frac{x^2 - a^2}{x - a}$.
7. If f is differentiable at a point a, then f is continuous at a.
8. Verify $f(x) = |x|$ by using mean value theorem on [-1, 2].
9. Calculate $\lim_{x\to\infty} \frac{\cos x - 1}{x^2}$ by L-Hospital's Rule.
10. If $f(x) = x$ on [0,1] and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$, find the upper and lower Darboux sums?
11. Show that $\left|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)dx\right| \leq \frac{16\pi^3}{3}$.
12. Calculate $\lim_{x\to 0} \frac{1}{x}\int_{-2\pi}^{2\pi} e^{t^2} dt$.

Section – B

II. *Answer the following questions.* [4 x 12 = 48]

13. (a) Determine limit of the sequence $< s_n > = < \frac{n}{n^2+1} >$ and prove your claim.

[OR]

(b) Prove that every sequence (s_n) has a monotonic subsequence.

14. (a) If f is continuous at x_0 and g is continuous at $f(x_0)$ then the composite function gof is continuous at x_0.

[OR]

(b) Show that $f(x) = \frac{1}{x^2}$ is not uniformly continuous on $(0, \infty)$ or even on the set (0,1).

15. (a) State and prove Lagrange's mean value theorem.

[OR]

(b) Discuss the differentiability of $f(x) = |x| + |x - a|$ in R.

16. (a) Prove that every continuous function f on [a,b] is integrable.

[OR]

(b) State and prove Fundamental theorem of calculus.

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NIZAM COLLEGE (AUTONOMOUS)

FACULTY OF SCIENCE

B.SC. IV- SEMESTER EXAMINATIONS, MAY – 2023

MATHEMATICS : PAPER - IV

(ALGEBRA)

TIME: 3 HOURS MAX. MARKS: 80

SECTION – A

I. *Answer any EIGHT of the following questions.* [8 x 4 = 32]

1. Define sub group. Find all sub groups of Z_{30}.
2. For any two elements a,b a group G prove that $(ab)^{-1} = b^{-1}a^{-1}$.
3. Let G be a group and H,K be two subgroups of G. Then show that $K = \{hk/h \in H, k \in K\}$.
4. Find the order of the following disjoint cycles $(i)(1\ 2\ 3)(4\ 5\ 6)(7\ 8), (ii)(1\ 4\ 3\ 2)\ (5\ 6)$.
5. Define cyclic group. Give an example.
6. Define permutation express $f = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,1\,2\,4\,6\,5 \end{pmatrix}$ as a product of transpositions.
7. Prove that every subgroup of an abelian group is normal.
8. Determine all group homomorphism from Z_{12}, Z_{30} .
9. Define Homomorphisim and Isomorphisim .
10. Define that following : (i) Principal Ideal (ii) Maximal Ideal.
11. Define characteristic of ring.
12. Is the ring 2Z is isomorphic to ring 3Z.

SECTION – B

II. ***Answer the following questions using internal choice.*** [4 x 12 = 48]

13. (a) Define Group. Prove that the set G $\{1,2,3,4,5,6\}$ is a abelian Group with respect to $\times_7$

[OR]

(b) Let $\alpha, \beta \in S_6$ and $\alpha = (124536)$; $\beta = (143256)$ then evaluate $\alpha\beta, \alpha\beta^{-1}, \alpha^{-1}$.

14. (a) State and prove Lagrange's Theorem.

[OR]

(b) State and prove Cayle's Theorem.

15. (a) Define Normal subgroup. Prove that A subgroup H of a group G is normal. If $xHx^{-1} = H\ \forall\ x \in G$.

[OR]

(b) State and prove Fundamental theorem an Homomorphism of groups.

16. (a) Prove that intersection of two ideals of a ring R is a ideal of R.

[OR]

(b) Prove that every finite integral domain is a field.

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CODE NO. 23M431/NC/MAT-4(B/L)

NIZAM COLLEGE (AUTONOMOUS)

FACULTY OF SCIENCE

B.Sc. IV- SEMESTER EXAMINATIONS, MAY – 2023

MATHEMATICS : PAPER – 4

(ALGEBRA)

TIME : 2 HOURS MAX. MARKS : 40

SECTION-A

I. Answer ALL questions. **(4x3=12)**

1. Prove that identity element is unique in the group.
2. Find the left cosets of the subgroup $4Z$ of Z.
3. Find the zero divisors in Z_6.
4. If R is a unity and U is an ideal of R where $1 \in U$, then prove that $U = R$.

SECTION-B

II. Answer the following questions using internal choice. **(4x7=28)**

5. (a) Show that the set $\{1, -1, i, -i\}$ is an abelian group with respect to usual multiplication.

   (OR)

   (b) If a cyclic group G is generated by an element a of order n *then* a^m is a generator of G. if and only if m and n are relatively prime.

6. (a) Show that intersection of two normal subgroups is a normal subgroup.

   (OR)

   (b) If G is a finite group and H is the subgroup of G then prove that $O\ (H)/O\ (G)$.

7. (a) Prove that $Q[\sqrt{2}] = \{a + b\sqrt{2} \,/\, a, b \in Q\}$ is an integral domain with respect to usual addition and multiplication.

   (OR)

   (b) Let R be a commutative ring with unity and let A be an ideal of R. Then $\frac{R}{A}$ is a field if and only if A is maximal ideal.

8. (a) Prove that every ideal of a ring R is the kernel of a ring homomorphism of R.

   (OR)

   (b) Let $R_1 R^1$ be two rings and $f: R \to R^1$ be homomorphism with Kernel U then prove that $\bar{R}$ is isomorphic to $\frac{R}{U}$

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Code No. 23J531 /NC/MAT

Nizam College (Autonomous)

Faculty of Science

B.SC. V- Semester Examinations, January - 2023

Mathematics : Paper-V

Time : 3 Hours Max. Marks : 80

Section – A

I. *Answer any EIGHT of the following questions.* [8 x 4 = 32]

1. Define linear independent set, dependent set in a vector space.
2. Prove that intersection of two subspaces is again a subspace.
3. Define $T: P_2 \to R^2$ by $T(p) = \begin{bmatrix} p(0) \\ P(1) \end{bmatrix}$ then prove that T is linear transformation.
4. Define a characteristic equation of a matrix A. Find the characteristic equation of $A = \begin{bmatrix} 1 & 4 \\ 5 & 0 \end{bmatrix}$.
5. If A is a 4×7 matrix with 4-dimentional nul space then find rank of A.
6. The Characteristic polynomial of 6x6 matrix is $\lambda^6 - 4\lambda^5 - 12\lambda^4$. Find eigen values and their multiplicities.
7. Define diagonalizable of matrix. Prove that $A = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable.
8. Find the eigen values and a basis for each eigen space in C^2 for $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$.
9. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, u = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, v = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ are u and v eigen vectors of A?
10. Define orthonormal set. Prove that the set $\{u_1, u_2, u_3\}$ is orthonormal where $u_1 = \left[\frac{3}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{3}{\sqrt{11}}\right]$, $u_2 = \left[\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]$ and $u_3 = \left[\frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}}\right]$.
11. In an inner product space, any orthogonal set of non-zero vectors is linearly independent.
12. u, v two vectors are orthogonal if and only if $\|u - v\|^2 = \|u\|^2 + \|v\|^2$.

[P.T.O]

Section – B

II. Answer the following questions using internal choice. [4 x

13 (a) (i) Prove that nul A is subspace of R^n, where A is $m \times n$ matrix.

(ii) Prove that col A is subspace of R^m, where A is $m \times n$ matrix.

[OR]

(b) (i) Find basis for the nul space of given matrix $\begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 1 & 4 \\ 3 & -1 & -7 & 3 \end{bmatrix}$

(ii) Define a coordinate vector in R^n and find the coordinate vector

X relative to β is a standard basis in R^3 and $X = [-1, \quad -3, \quad 5]$

14. (a) State and prove Rank Theorem.

[OR]

(b) Let $b_1 = [1 \quad -3]$, $b_2 = [-2 \quad 4]$, $c_1 = [-7 \quad 9]$, $c_2 = [-5 \quad 7]$ be vectors in R^2. If B , C are bases for R^2, where $B = \{b_1, b_2\}$, $C = \{c_1,$

Then find (i) change of coordinate matrix from C to B and (ii) change coordinate matrix from B to C.

15. (a) Prove that $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ is diagonalizable then find P for A with PA where P is some invertible matrix and D is diagonal matrix. and using Compute A^4.

[OR]

(b) Diagonalise the matrix $\begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ if possible.

16. (a) (i) In an inner product space V(F), $|\langle u, v \rangle| \leq \|u\| \|v\|$ for all $u, v \in V$

(ii) In an inner product space V(F), $\|u + v\| \leq \|u\| + \|v\|$ for all $u, v \in V$

[OR]

(b) If the set $\{u_1, u_2, u_3\}$ is a basis of R^3 then construct an orthonormal basis for R^3. Where $u_1 = [1, \quad 1, \quad 1]$, $u_2 = [1, \quad 1, \quad 0]$ and $u_3 = [1, \quad 0, \quad 1]$.

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CODE NO. 23M531/NC/MAT-B/L

NIZAM COLLEGE (AUTONOMOUS)
FACULTY OF SCIENCE
B.SC. V- SEMESTER EXAMINATIONS, MAY – 2023
MATHEMATICS: PAPER - V
(LINEAR ALGEBRA)

TIME: 2 HOURS MAX. MARKS: 40

SECTION-A

I. Answer All Questions. **(4x3=12)**

1. Let V be the first quadrant in the xy-plane; Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0, y \geq 0 \right\}$. If u and v are in V, is $u+v$ in V? Why?
2. Determine if $\{v_1, v_2, v_3\}$ is linearly dependent or linearly independent where $v_1 = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$.
3. Is $\lambda = 2$ an eigen value of $\begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$? Why or why not?
4. Define inner product.

SECTION-B

II. Answer the following Questions using internal choice. **(4x7=28)**

5. (a) i) Show that intersection of two subspaces is again a subspace.

ii) Let H be set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector v in R^3 such that $H =$ span $\{v\}$.

(OR)

(b) Find a spanning set for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

6. (a) Prove that an indexed set $\{v_1, v_2, ...v_p\}$ of two or more vectors with $v_1 \neq 0$ is linearly dependent iff ∃ some $v_j (j > 1)$ is linear combination of the preceding vectors $v_1, v_2, ...v_{j-1}$.

(OR)

(b) State and prove Rank theorem.

P. T. O

7. (a) Find the characteristic polynomial and the real eigen values of $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$.

(OR)

(b) Find basis for eigen space corresponding to the matrix $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ and eigen values $\lambda = 1,3$.

8. (a) Daigonalise the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$.

(OR)

(b) Assume the mapping $T: P_2 \to P_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$. Find the matrix representation of T relative to basis $B = \{1, t, t^2\}$.

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Code No. 23D/531 /NC/MAT

Nizam College (Autonomous)

Faculty of Science

B.SC. V- Semester Examinations, December - 2023

Mathematics : Paper-V

(Linear Algebra)

Hours Max. Marks : 80

Section – A

er any EIGHT of the following questions. [8 x 4 = 32]

1. Define null space. Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and let $\boldsymbol{u} = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$. Determine if $\boldsymbol{u}$ belongs to the null space of A.
2. Define subspace. Show that the set S=$\left\{\begin{bmatrix} -a+2b \\ a-2b \\ 3a-6b \end{bmatrix} : a, b \text{ in } \mathbb{R}\right\}$ is a subspace.
3. If $B = \{\overline{b_1}, \overline{b_2}\}$ and $C = \{\overline{c_1}, \overline{c_2}\}$ be bases for $\mathbb{R}^2$, find the change of coordinate matrix from B to C and C to B where $\overline{b_1} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, $\overline{b_2} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\overline{c_1} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $\overline{c_1} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.
4. Find the characteristic equation of $A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
5. $T: \mathbb{R}^2 \to \mathbb{R}^2$ Such that $T(\vec{x}) = A\vec{x}$. Find a basis B for $\mathbb{R}^2$ with the property $[T]_B$is diagonal where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$.
6. Find the eigen values and a basis for each eigen space in $\mathbb{C}^2$ if $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$.
7. Let $\boldsymbol{y} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $\boldsymbol{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of $\boldsymbol{y}$ onto $\boldsymbol{u}$ and write $\boldsymbol{y}$ as sum of two orthogonal vectors.
8. Define unit vector. Let $\boldsymbol{v} = (1, -2, 2, 0)$. Find a unit vector in the same direction of $\boldsymbol{v}$.
9. Show that the null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.
10. Let $A = PDP^{-1}$ then compute A^4 if $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.
11. Show that the eigen values of a triangular matrix are the entries on its main diagonal.
12. Show that the set $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is an orthogonal set if $\boldsymbol{u_1} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{u_2} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $\boldsymbol{u_3} = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$

[P.T.O]

Section – B

II. Answer the following questions. [4x]

13 (a) Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

[OR]

(b) Define Basis of a vector space. Find the dimension of the subspace

$$S = \left\{ \begin{bmatrix} a - 4b - 2c \\ 2a + 5b - 4c \\ -a + 2c \\ -3a + 7b + 6c \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}.$$

14. (a) State and prove Rank theorem. Find the rank of

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & -2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}.$$

[OR]

(b) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. An eigen value of A is 2. Find a basis for the corresponding eigen space.

15. (a) Determine if the following matrix is diagonalizable. $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

[OR]

(b) $T: \mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ is a linear transformat

(a) Find the B- matrix for T, when the B is the basis $\{1, t, t^2\}$.

(b) Verify that $[T(\boldsymbol{p})]_B = [T]_B[\boldsymbol{p}]_B$ for each $\boldsymbol{p}$ in $\mathbb{P}_2$.

16. (a) State and prove the Gram- Schmidt process.

[OR]

(b) The set $\{x_1, x_2, x_3\}$ is a basis for a subspace W of $\mathbb{R}^4$ where

$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Construct an orthogonal basis for W.

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CODE NO. 23M631/NC/MAT

NIZAM COLLEGE (AUTONOMOUS)

FACULTY OF SCIENCE

B.SC. VI- SEMESTER EXAMINATIONS, MAY – 2023

MATHEMATICS : PAPER - VI

(NUMERICAL ANALYSIS)

TIME: 3 HOURS

MAX. MARKS: 80

SECTION – A

I. *Answer any EIGHT of the following questions.* [8 x 4 = 32]

1. Evaluate the sum $S=\sqrt{3}+\sqrt{5}+\sqrt{7}$ to 4 significant digits and find its absolute and relative errors.
2. Define absolute, relative and percentage errors. Round off the given numbers 1.6583, 30.0567, 0.859378 and 3.14159 to four significant digits.
3. Explain Newton- Raphson method.
4. From the given data

x	0	1	2	3	4
$f(x)$	1	14	15	5	6

Find $f(3)$ using forward difference table.

5. Find $f(2.5)$ using the following table

x	1	2	3	4
$f(x)$	1	8	27	64

6. Find the cubic polynomial from the following values; $y(1) = 24, y(3) = 120, y(5) = 336$ *and* $y(7) = 720$.
7. A rod is rotating in a plane about one of its ends. The angle θ (in radians) at different times t (in seconds) are given below.

t	0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.15	0.50	1.15	2.0	3.20

Find its angular velocity when $t = 0.6$ seconds.

8. The following data gives the melting pointsof an alloy of lead and zinc

% of lead in the alloy (p)	50	60	70	80
Temperature (Q°C)	205	225	248	274

Find the melting point of the alloy containing 54% of lead using appropriate interpolation formula.

9. Evaluate $\int_1^7 \frac{1}{x}dx$ using simpson's $\frac{1}{3}$ rule.
10. Given $\frac{dy}{dx} - 1 = xy$ *and* $y(0) = 1$, obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to 4 decimal places.
11. Given $\frac{dy}{dx} = x + yx^4$ where $y(0) = 3$. Find $y(0.1)$ and $y(0.2)$ using Picard's method.
12. Given that $\frac{dy}{dx} = x^2 + y,\ y(0) = 1$ determine $y(0.02)$ *and* $y(0.04)$ using Euler's method.

[P.T.O]

SECTION – B

II. Answer the following questions using internal choice. [4 x 12 =

13. (a) Using Iteration method, find a real root of the equation $2x - 3 = cosx$ upto 4 decimal places with $x_0 = \frac{\pi}{3}$.

[OR]

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ using the method of False positi

14. (a) Using Lagrange's interpolation formula, find the value of $y(10)$ from the followin tables.

x	5	6	9	11
y	12	13	14	16

[OR]

(b) Using Stirling's formula find $cos(0.17)$ given that $cos(0) = 1, cos(0.05) = 0.9988, cos(0.10) = 0.9950, cos(0.15) = 0.9888, cos(0.20) = 0.9801, cos(0.25) = 0.9689$ and $cos(0.30) = 0.9553$.

15. (a) Find the values of a_0 *and* a_1 so that $Y = a_0 + a_1 x$ fits the data given in the table.

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

[OR]

(b) Using simpson's $\frac{3}{8}$ rule, evaluate $\int_0^1 \frac{1}{1+x} dx$ with $h = \frac{1}{6}$

16. (a) Compute the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$ using Taylor's series method for the solution of the problem $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

[OR]

(b) Given $\frac{dy}{dx} = 1 + y^2$ where y=0 when x=0 find $y(0.2), y(0.4)$ *and* $y(0.6)$ using Runge-Kutta fourth order method.

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CODE NO. 23M6831/NC/MAT-8-B/L

NIZAM COLLEGE (AUTONOMOUS)

FACULTY OF SCIENCES

B.Sc. VI - SEMESTER EXAMINATIONS-MAY-2023

MATHEMATICS - 8

(VECTOR CALCULUS)

Time: 2 HOURS] [Max.Marks=40

SECTION-A

I. Answer All Questions. (4x3=12)

1. Evaluate the line integral of the Vector field $\overline{u}=(xy,z^2,x)$ along the curve given by $x=1+t, y=0, z=t^2, 0\le t\le 3$.
2. Evaluate the surface integral of $u=(xy,x,x+y)$ over the surface S defined by $z=0$, $0\le x\le 1, 0\le y\le 2$ with the normal n directed in the positive direction.
3. Evaluate the line integral $\int_c F.dr$ where $F=(5z^2,2x,x+2y)$ and the curve C is given by $x=t, y=t^2, z=t^2, 0\le t\le 1$
4. Find the volume integral of the scalar field $\phi=x^2+y^2+z^2$ over the region V specified by $0\le x\le 1, 1\le y\le 2, 0\le z\le 3$.

SECTION-B

II. Answer all of the following questions using internal choice. (4x7=28)

5. (a) Find the circulation vector of the vector $F=(y,-x,0)$ along the curve consisting of the two straight line segments $a)\ y=1, 0\le x\le 1$ $b)\ x=1, 1\le y\le 2$.

(OR)

(b) If S is the entire x,y-plane, evaluate the integral $I=\iint_S e^{-x^2-y^2}ds$, by transforming the integral in to polar co-ordinates.

6. (a) By considering the line integral of $F=(y,x^2-x,0)$ around the square in the x,y-plane connecting the four points $(0,0),(1,0),(0,1)$ and $(1,1)$, show that F cannot be a conservative vector field.

(OR)

(b) Find the volume of the tetrahedron with vertices at $(0,0,0),(a,0,0),(0,b,0)$ and $(0,0,c)$.

7. (a) Show that the gradient of the scalar field $\phi=r=|\overline{r}|$ is $\frac{|\overline{r}|}{r}$ and interpret this result geometrically.

(OR)

(b) Show that the vector field $F=(2x+y,x,2z)$ is conservative.

8. (a) For $\phi(x,y,z)=x^2+y^2++z^2+xy-3x$, find $\nabla\phi$ and find the minimum value of ϕ.

(OR)

(b)For what values, if any, of the constants a and b is the vector field $u=(y\cos x+axz, b\sin x+z, x^2+y)$.

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CODE : 23M6731/NC/MAT-7 (B/L)

NIZAM COLLEGE (AUTONOMOUS)

FACULTY OF SCIENCES

B.Sc., VI SEMESTER EXAMINATIONS-MAY- 2023

MATHEMATICS PAPER-VII NUMERICAL ANALYSIS

[Time: 2 HOURS] [Max.Marks=40]

SECTION A

I. Answer ALL Questions. (4x3=12)

1. Describe about the Bisection method.
2. Define Absolute error and Relative error.
3. Determine the linear Lagrange interpolating polynomial that passes through the points $(2,4)$ and $(5,1)$.
4. Approximate $\int_0^{1.4} \frac{2x}{x^2-4}dx$ using Trapezoidal rule.

SECTION-B

II. Answer the following Questions using internal choice. (4x7=28)

5. (a) Use a fixed point iteration method to determine a solution accurate to within 10^{-2} for $x^4-3x^2-3=0$.

[OR]

(b) Use Newton's method to determine a solution for $f(x)=\cos x - x = 0$.

6. (a) Use Secant method to find solution in $[0.1,1]$ accurate to within 10^{-4} for $600x^4-550x^3+200x^2-20x-1=0$.

[OR)]

(b) Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$ for given $x_0=1, x_1=1.25$ and $x_2=1.6$ where $f(x)=\log_{10}(3x-1)$.

7. (a) Use Neville's Method to obtain the approximation for Lagrange's interpolating polynomial of degree one, two and three to approximate of $f(0.25)$ if $f(0.1)=0.62049958, f(0.2)=-0.28398668, f(0.3)=0.00660095, f(0.4)=0.24842440$.

[OR]

(b) Use Newton forward difference formula to construct interpolating polynomials of degree three for the following data. Approximate the specified value using each of the polynomials $f(0.43)$ if $f(0)=1, f(0.25)=1.64872, f(0.5)=02.71828, f(0.75)=4.48169$.

8. (a) Approximate the integral $\int_1^{1.5} x^2 \ln x\, dx$ by using simpson's rule.

[OR]

(b) Use Romberg integration to compute $R_{3,3}$ for the integral $\int_{-1}^{1}(\cos x)^2\, dx$.

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